Çankaya University
Department of Computer Engineering
2014 - 2015 Fall Semester

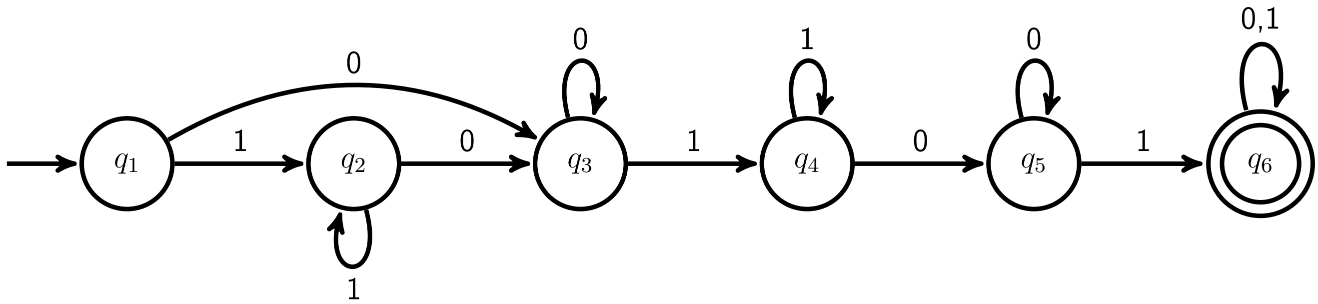
CENG 491 - Formal Languages and Automata

First Midterm Examination

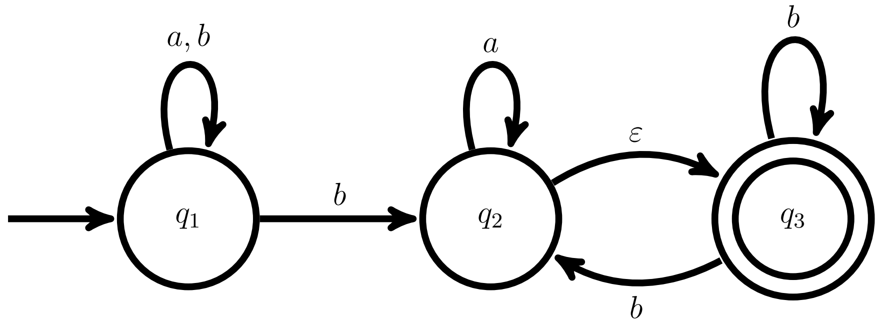
1) Give the state diagram of a DFA that recognizes the language A over alphabet $\Sigma = \{0, 1\}$ where

$$A = \{w \mid w \text{ contains } 1111 \text{ or } 0000\}$$

2) The following DFA recognizes the language B over alphabet $\Sigma = \{0, 1\}$. Describe B verbally.



3) Convert the following NFA to a DFA:

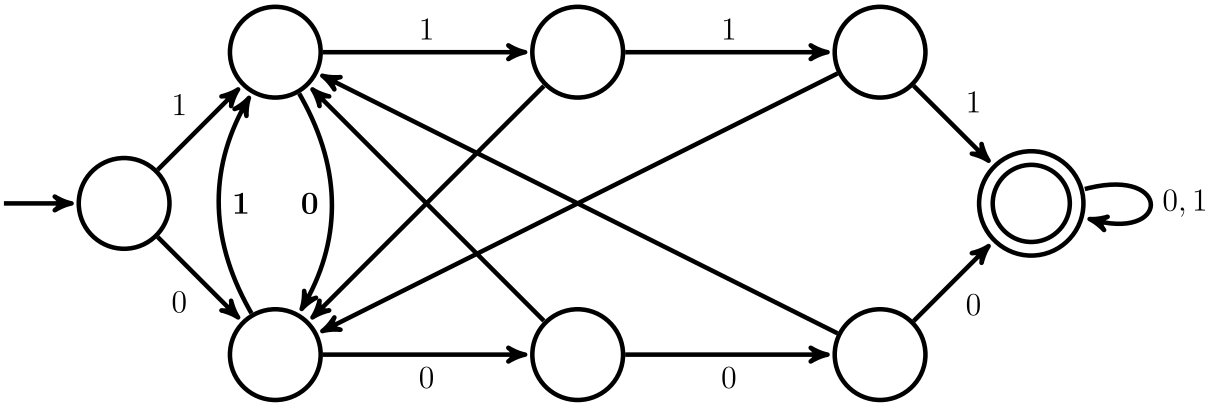


4) For the regular expression $\left[(bb \cup aba)^* baa\right]^*$, find an equivalent NFA.

5) Show that the language $A = \{a^{2n} b^{n+2} c^{n-2} \mid n \geqslant 2\}$ is not regular.

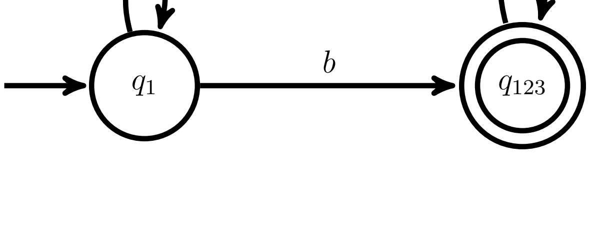
Answers

1)

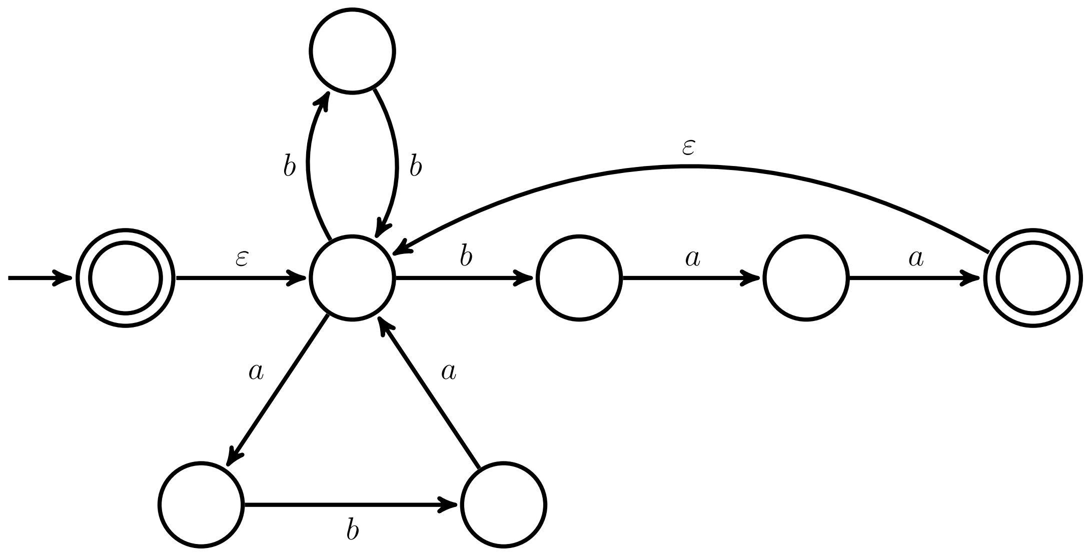


2) Contains at least two 01's
OR
If it starts with zero, changes symbols at least 3 times, if it starts with 1, changes symbols at least 4 times.

3)



4)



5) Assume A is regular. Let pumping length be p.

Consider

$$s = a^{2p}b^{p+2}c^{p-2}$$

According to pumping lemma, we can find x, y, z such that $s = xyz$. We also know that $|xy| \leqslant p$ therefore y consists of $a's$ only. Suppose $y = a^k$. In this case

$$xy^i z = a^{2p+ik}\, b^{p+2}\, c^{p-2} \notin A$$

We have a contradiction, so A is not regular.

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2014 - 2015 Fall Semester

CENG 491 - Formal Languages and Automata
Second Midterm Examination

1) Find a context free grammar that recognizes the language over $\Sigma = \{0, 1\}$ consisting of strings of odd length where first, middle and last symbols are the same.

2) Find a PDA that recognizes the language $\{a^n b^m \mid m > n\}$.

3) Consider the following languages over $\Sigma = \{a, b, c, d\}$:

- $\{a^n b^m c^m d^n\}$
- $\{a^n b^n c^m d^m\}$
- $\{a^n b^m c^n d^m\}$

a) Which one is non-context free?

b) Show that it is not using pumping lemma.

4) Describe a Turing Machine deciding the language $A = \{0^n 1^{n^2} \mid n \geqslant 1\}$.

5) What language does the following Turing machine recognize? Is it a decider?
(Input alphabet is: $\Sigma = \{a, b\}$)

1. Sweep from left to right. IF there is any a after the first b, REJECT.
2. Move head to start. Search for a.
   IF found, cross it. (Replace by $\times$)
   ELSE, Go to 6.
3. Search for b.
   IF found, cross it.
   ELSE, REJECT.
4. Search for b.
   IF found, cross it.
   ELSE, REJECT.
5. Go to 2.
6. Move head to start. Search for b.
   IF found, ACCEPT.
   ELSE, REJECT.

Answers

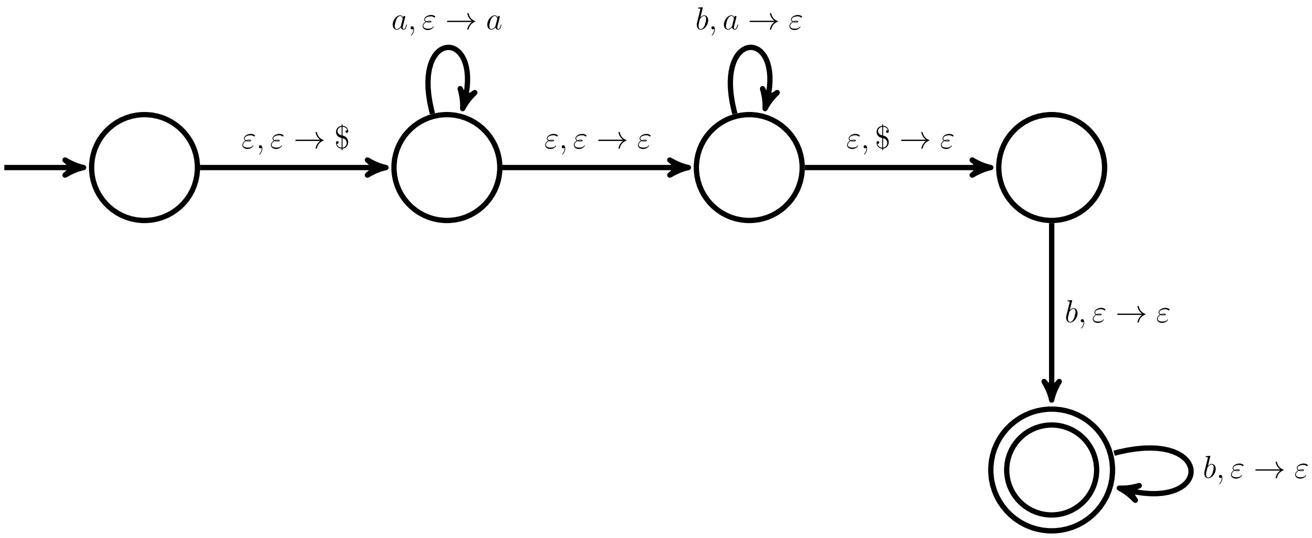
1)

$S \to 0A0 \mid 1B1 \mid 0 \mid 1$
$A \to 0A0 \mid 1A1 \mid 0A1 \mid 1A0 \mid 0$
$B \to 0B0 \mid 1B1 \mid 0B1 \mid 1B0 \mid 1$

OR

$S \to 0A0 \mid 1B1 \mid 0 \mid 1$
$A \to CAC \mid 0$
$B \to CBC \mid 1$
$C \to 0 \mid 1$

2)



3) a) $\{a^n b^m c^n d^m\}$

b) Suppose the language is context free. Let p be the pumping length. Choose s as $s = a^n b^m c^n d^n$ where $m, n > p$. According to pumping lemma, we can find v, y such that

$$a^n b^m c^n d^n = uvxyz$$

If v or y contain more than one type of symbol, clearly $uv^2xy^2z \notin A$ because symbols are out of order. Therefore v and y can consist of a single symbol only.

In this case, we have two choices. v must consist of $a's$ and y must consist of $c's$, or v must consist of $b's$ and y must consist of $d's$. Both cases violate the rule $|vxy| < p$

Therefore we cannot pump this string so the language is not context free.

4)

1. Sweep from left to right. IF there is no 0, or no 1, or if they are out of order, REJECT.

2. Move head to start. Search for 0.
   IF found,
   Mark it.
   Move right until blank. Write #.
   Go to 2.
   //Write as many #'s as there are 0's.

3. Search for #.
   IF found
   Mark it.
   Unmark all marked 0's.
   Shuttle between 0's and 1's. Mark one 1 for each one 0.
   IF 0 is not found, Go to 3.
   IF 1 is not found, REJECT.

4. ELSE (# not found)
   Move head to start. Search for 1.
   IF found, REJECT.
   ELSE, ACCEPT.

5) It is a decider, it can not enter any infinite loops.
Its language is: $\{a^n b^m \mid m \geqslant 2n+1\}$.

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CENG 491 - Formal Languages and Automata

Final Examination

1) Give the state diagram of a DFA that recognizes the language A over alphabet $\Sigma = \{0, 1\}$ where $|w| \geqslant 2$ and the first two and the last two digits of w are identical.
For example:
$10010010 \in A, 0011100 \in A$ but
$001 \notin A, 100001 \notin A$

2) Convert the following grammar into Chomsky normal form:
$S \to AbA$
$A \to aB \mid bBb \mid \varepsilon$
$B \to A \mid ba$

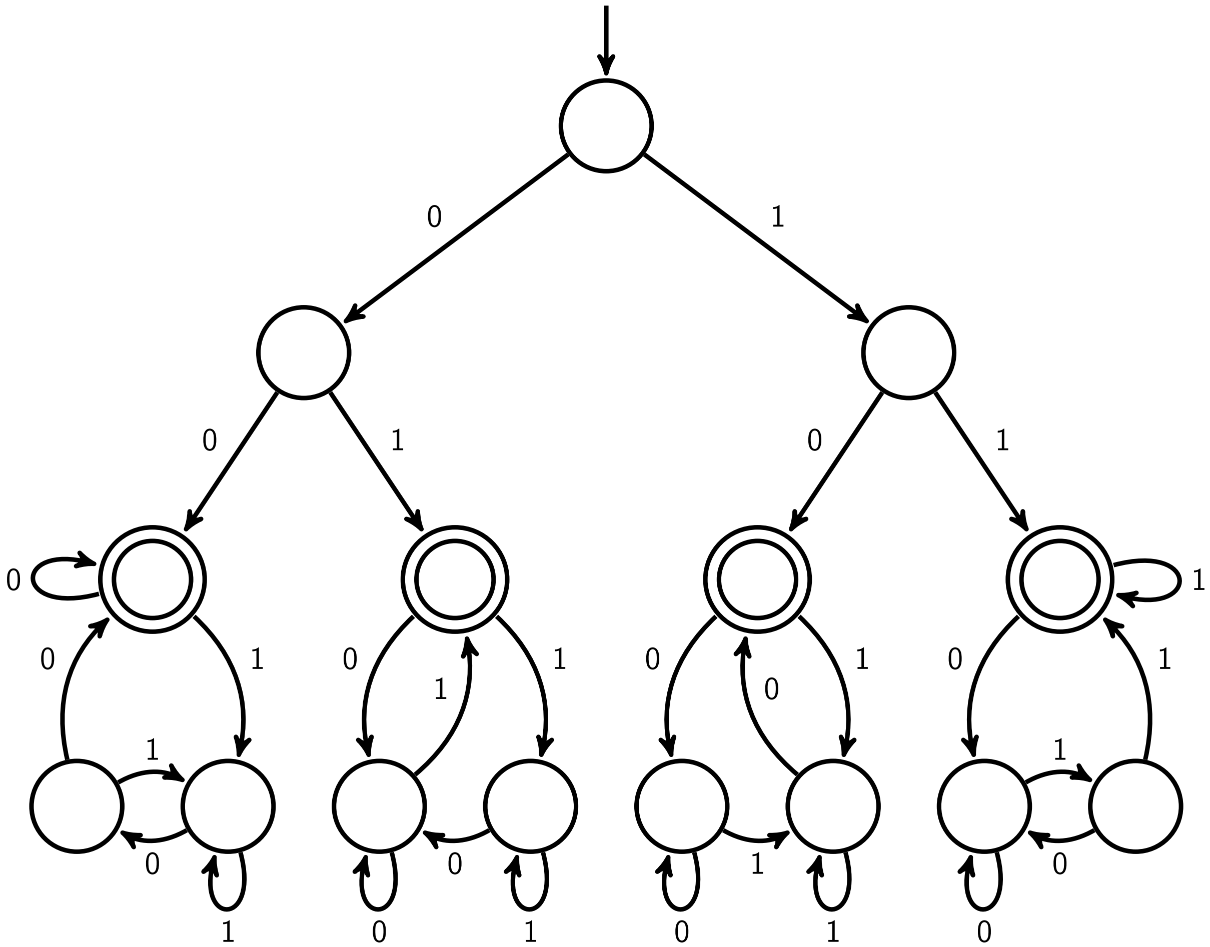
3) Let A be the language in $\{0, 1\}^*$ made of strings where the number of zeros is at least 3 times the number of ones.
Describe a Turing Machine recognizing A.

4) You are given a set of n distinct positive integers. You want to determine if there are two integers p, q in the set such that $p = q^2$.
Write an algorithm in pseudo-code for this problem. Show that it is in P.

5) Consider the following problem:
Given a graph, is there a way to partition the vertices into 4 subsets such that no two elements in a subset are connected?
Show that this problem is in NP.

Answers

1)



2) First eliminate $B \to A$, then eliminate $A \to \varepsilon$ and then break triples to obtain:

$S \to YA \mid AY \mid AC \mid b$
$A \to XB \mid YD \mid XA \mid YE \mid YY \mid a$
$B \to YX$
$C \to YA$
$D \to BY$
$E \to AY$
$X \to a$
$Y \to b$

3)

1. Move head to start. Search for 1.
   IF found, cross it. (Replace by $\times$)
   ELSE, ACCEPT.

2. Repeat 3 times:
   Move head to start. Search for 0.
   IF found, cross it. (Replace by $\times$)
   ELSE, REJECT.

3. Go to 1.

4)

INPUT Integer $A[1], A[2], \ldots, A[n]$
For $i = 1$ to n
    For $j = 1$ to n
        If $A[i] \; * A[i] \; == \; A[j]$
            Return TRUE
        EndIf
    EndFor
EndFor
Return FALSE

This algorithm clearly does $\Theta(n^2)$ operations, so it is in P.

5) Suppose there are n vertices. Then, there are at most $\dfrac{n(n-1)}{2} = \Theta(n^2)$ edges.

We can check a given solution by checking all edges one by one. (We will return FALSE if the two vertices connected by the edge belong to the same subset)

Therefore a given a solution can be verified in $\Theta(n^2)$ operations. The problem is in NP.

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CENG 491 - Formal Languages and Automata

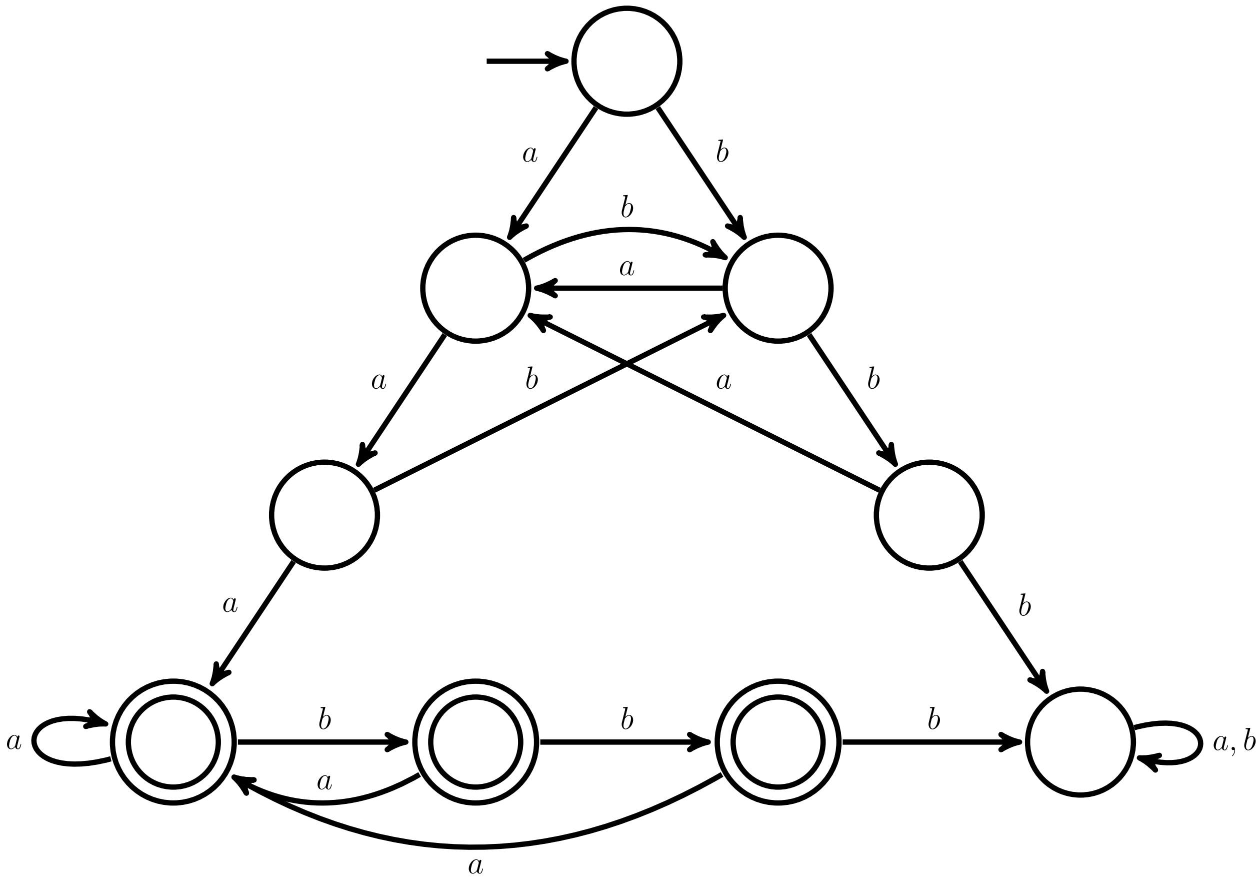
Name-Surname: 01.10.2014

ID Number:

CLASSWORK 1

Give the state diagram of a DFA that recognizes the language A over alphabet $\Sigma = \{a, b\}$ where $A = \{w \mid w \text{ contains } aaa \text{ but does not contain } bbb\}$.

Answer:



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CENG 491 - Formal Languages and Automata

Name-Surname: 02.10.2014

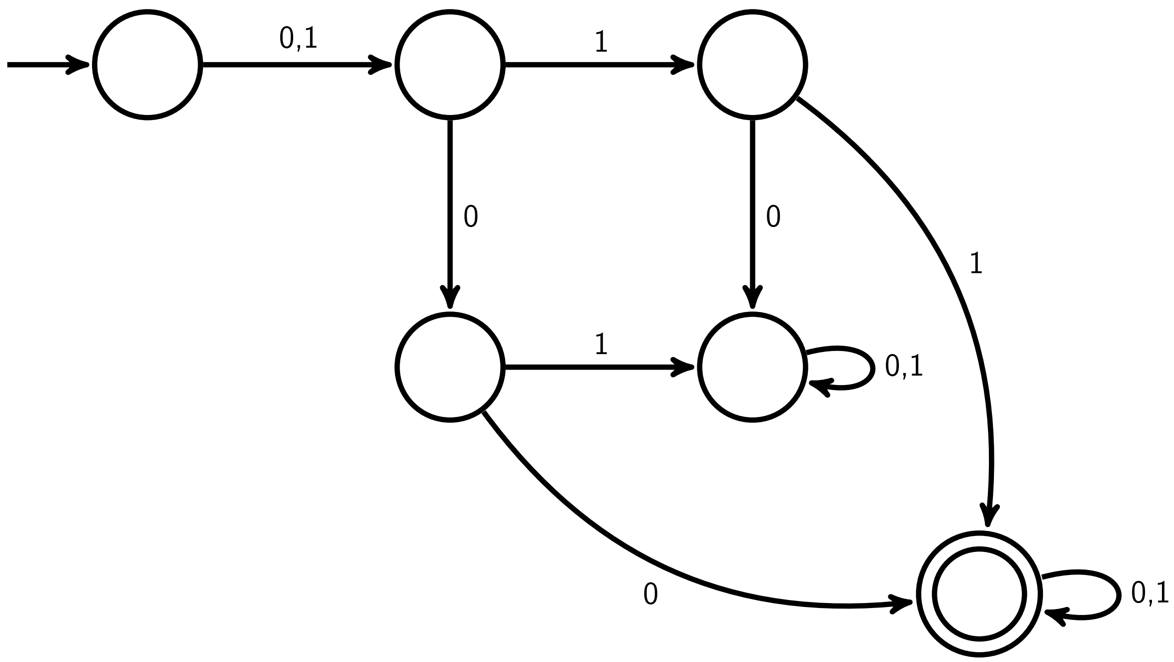
ID Number:

CLASSWORK 1

Let:
$A = \{$Strings of length at least 3 whose second and third items are the same$\}$ over alphabet $\Sigma = \{0, 1\}$.

Give the state diagram of the DFA that recognizes this language.

Answer:



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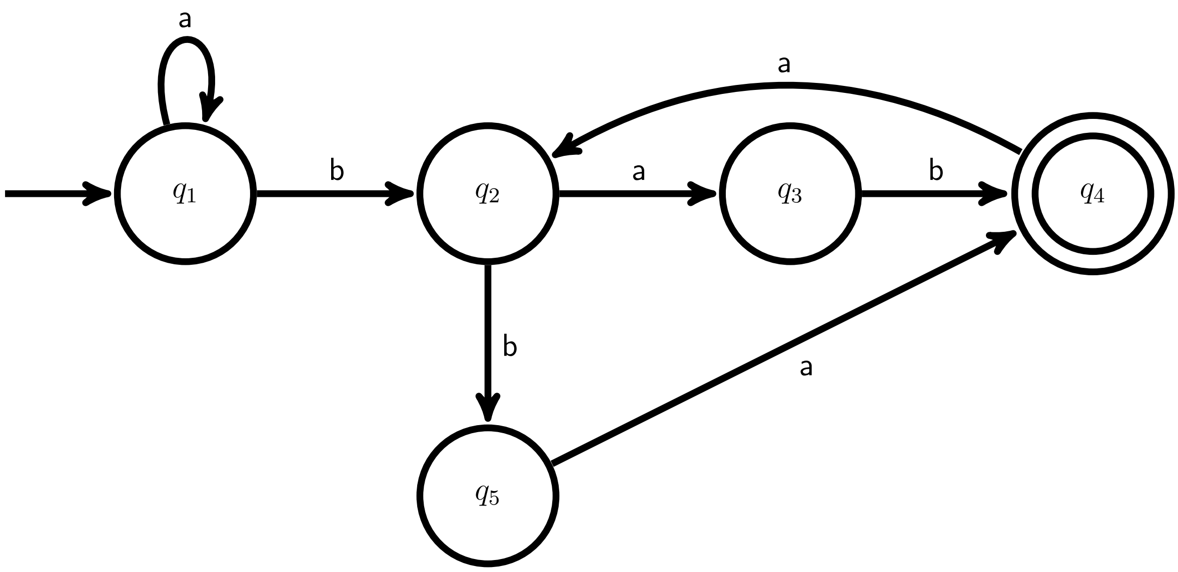
CENG 491 - Formal Languages and Automata

Name-Surname: 15.10.2014

ID Number:

CLASSWORK 2

Find a regular expression equivalent to the language recognized by the following NFA:



Answer:

$$a^*b\big(aba \cup baa\big)^*\big(ab \cup ba\big)$$

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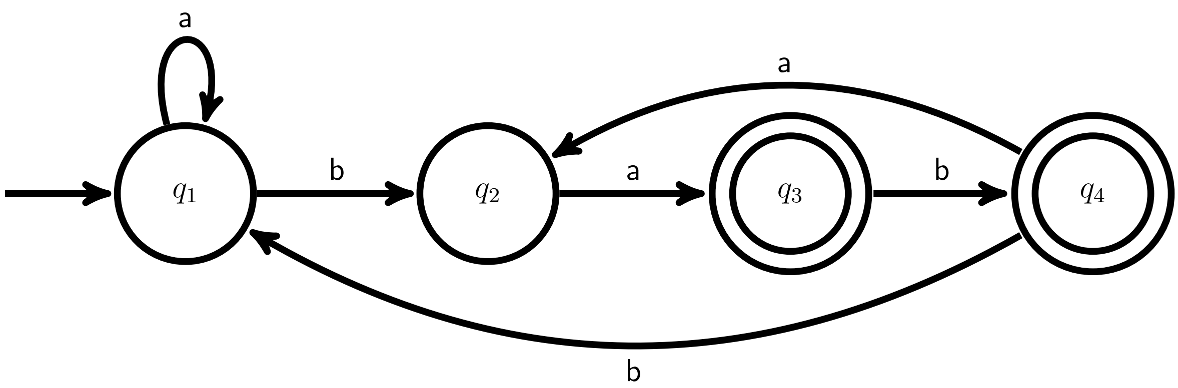
Name-Surname: 16.10.2014

ID Number:

CLASSWORK 2

Find a regular expression equivalent to the language recognized by the following NFA:



Answer:

$$\left[a \cup b(aba)^* abb\right]^* b\big(aba\big)^* \big(a \cup ab\big)$$

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CEng 491 - Formal Languages and Automata

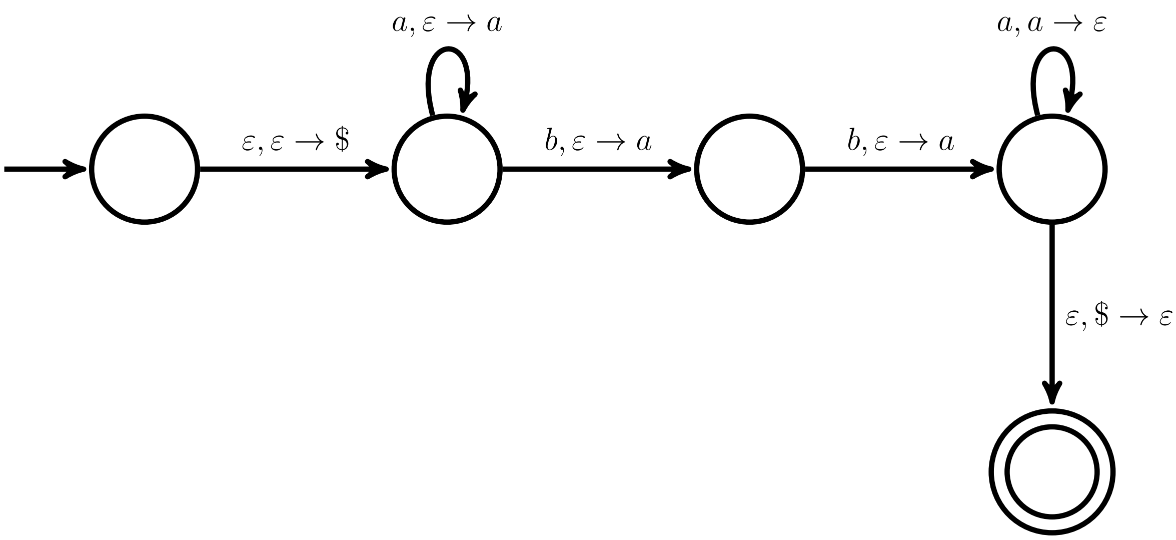
Name-Surname: 05.11.2014

ID Number:

CLASSWORK 3

a) Find a PDA that recognizes the language $\{a^n b^2 a^{n+2} \mid n \geqslant 0\}$
b) Find a CFG generating the same language.

Answer:



$S \to aSa \mid b^2a^2$

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CENG 491 - Formal Languages and Automata

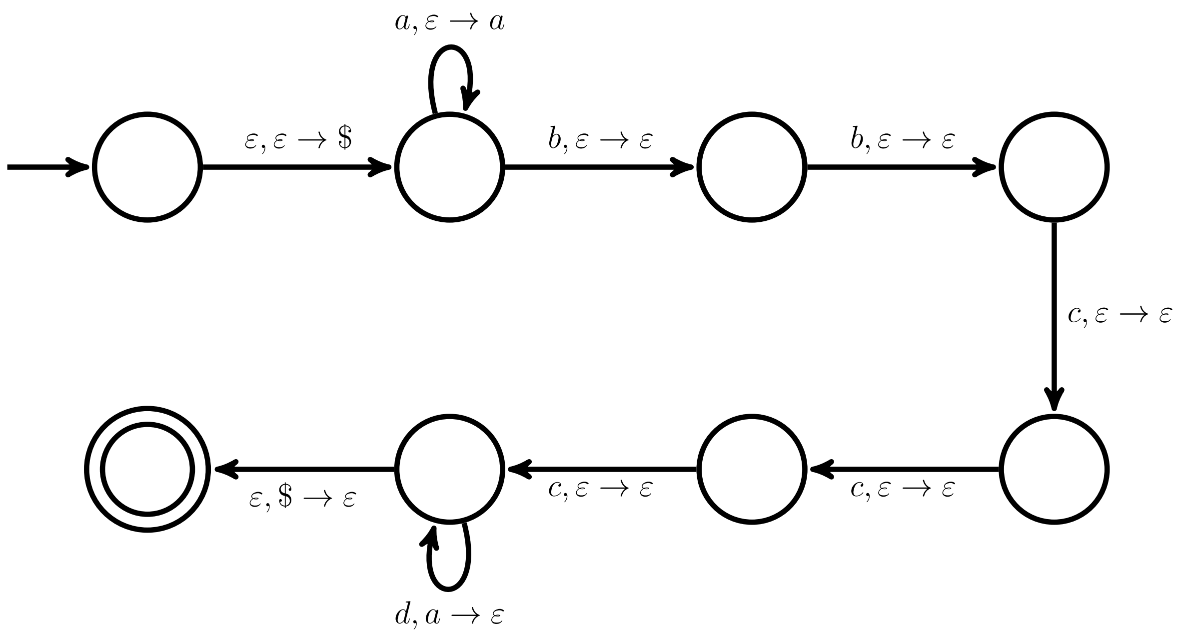
Name-Surname: 06.11.2014

ID Number:

CLASSWORK 3

a) Find a PDA that recognizes the language $\{a^n b^2 c^3 d^n \mid n \geqslant 0\}$
b) Find a CFG generating the same language.

Answer:



$S \to aSd \mid b^2c^3$

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CENG 491 - Formal Languages and Automata

Name-Surname: 12.11.2014

ID Number:

CLASSWORK 4

Use the pumping lemma to show that the following language is not context free:

$A = \{a^n bca^{4n} ba^n \mid n \geqslant 0\}$

Answer:

Suppose A is context free. Let p be the pumping length. Choose s as $s = a^p bca^{4p} ba^p$. According to pumping lemma, we can findv, y such that

$$a^p bca^{4p} ba^p = uvxyz$$

If v or y contain b or c, clearly $uv^2xy^2z \notin A$ because it is not in the given format. Therefore v and y can consist of $a's$ only.

But in this case, we can pump at most two of the parts including a. The size of the third part will remain the same, so pumped string will not be in A. For example, $v = a, y = aaaa$ will result in

$$uv^2xy^2z = a^{p+1} bca^{4p+4} ba^p \notin A$$

Therefore we cannot pump this string and A is not context free.

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Name-Surname: 13.11.2014

ID Number:

CLASSWORK 4

Use the pumping lemma to show that the following language is not context free:

$A = \{a^n b^m c^{2n} d^{2m} \mid m, n \geqslant 0\}$

Answer:

Suppose A is context free. Let p be the pumping length. Choose s as $s = a^n b^m c^{2n} d^{2m}$ and $n, m \geqslant p$. According to pumping lemma, we can find v, y such that

$$a^n b^m c^{2n} d^{2m} = uvxyz$$

If v or y contain more than one type of symbol, clearly $uv^2xy^2z \notin A$ because it is not in the given format. Therefore v and y can consist of a single symbol only.

If v consists of $a's$, then y must consist of (twice as many) $c's$. If v consists of $b's$, then y must consist of (twice as many) $d's$.

In bothe cases, $|vxy| > p$ and the third condition of pumping lemma is violated.

Therefore we cannot pump this string and A is not context free.

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Name-Surname: 19.11.2014

ID Number:

CLASSWORK 5

Describe a Turing Machine that gives the output $x_m, x_{m-1}, \ldots, x_2, x_1$ for the input $x_1, x_2, \ldots, x_{m-1}, x_m$.

Answer:

1. Move to Start. Find the first element that is not crossed. Call it a. Cross it.
   If not found, Go To 4.
2. Move right until meeting blank or cross. Move left. Read the element. Call it b.
3. Swap a and b if they are different. Cross a and b. Go To 1.
4. Sweep from left to right. Restore all crossed elements. Return.

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CEN G 491 - Formal Languages and Automata

Name-Surname: 20.11.2014

ID Number:

CLASSWORK 5

Describe a Turing Machine that gives the output $x_1, x_2, \ldots, x_m, x_1, x_2, \ldots, x_m$ for the input $x_1, x_2, \ldots, x_m$.

Answer:

1. Move to Start. Find the first element that is not crossed. Cross it.
   If not found, Go To 4.
2. Move right until meeting blank.
3. Write the element found in 1. Cross it. Go To 1.
4. Sweep from left to right. Restore all crossed elements. Return.

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Name-Surname: 26.11.2014

ID Number:

CLASSWORK 6

Let A be the language in $\{0,1\}^*$ made of strings containing an equal number of 0's and 1's.

Describe a Turing Machine recognizing A.

Is it a decider?

Answer:

1. Move head to start. Search for 0.
2. IF there's a 0,
   Cross it.
   Move head to start.
   Search for 1.
   IF there is a 1,
      Cross it.
      Go to 1.
   ELSE
      REJECT.
3. IF there's no 0,
   Move head to start.
   Search for 1.
   IF there is a 1,
      REJECT.
   ELSE
      ACCEPT.

This machine stops after finitely many steps, because loops are repeated (at most) as many times as the number of symbols in the input. It is a decider.

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Name-Surname: 27.11.2014

ID Number:

CLASSWORK 6

Let B be the language in $\{0,1\}^*$ made of strings containing more 0's than 1's.
Describe a Turing Machine recognizing B.
Is it a decider?

Answer:

1. Move head to start. Search for 0.
2. IF there's a 0,
   Cross it.
   Move head to start.
   Search for 1.
   IF there is a 1,
      Cross it.
      Go to 1.
   ELSE
      ACCEPT.
3. IF there's no 0,
   REJECT.

This machine stops after finitely many steps, because loops are repeated (at most) as many times as the number of symbols in the input. It is a decider.

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CENG 491 - Formal Languages and Automata

Name-Surname: 10.12.2014

ID Number:

CLASSWORK 7

Let A be the set of all 2×2 matrices with entries from $\mathbb{Q}$. Show that A is countable.

Answer:

We know that $\mathbb{Q}$ is countable. Suppose a list of $\mathbb{Q}$ is $\{q_1, q_2, \ldots\}$. We can count $\mathbb{Q} \times \mathbb{Q}$ by making an infinite table and counting diagonally.

Suppose we obtain a list $S = \{s_1, s_2, \ldots\}$. Now make a table for $S \times S$. The list will give elements of $A = \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$.

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Name-Surname: 11.12.2014

ID Number:

CLASSWORK 7

Let B be the set of all finite strings based on the alphabet $\{0, 1, \ldots, 9\}$. Show that B is countable.

Answer:

The following gives a list of all finite strings that can be written with this alphabet:

String
0
1
$\vdots$
9
00
01
$\vdots$
99
000
$\vdots$

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Department of Computer Engineering
CENG 491 - Formal Languages and Automata

Name-Surname: 24.12.2014

ID Number:

CLASSWORK 8

STRING MATCHING:

You are given a string of length n. You are also given a word (a second string) of length k, where $k \leqslant n$. You want to determine if the word occurs in the string.

Show that the string matching problem is in P.

Answer:

```
INPUT String A[n], Word B[k]
For i = 1 to n − k + 1
    test = TRUE
    For j = 1 to k
        If A[i + j − 1] ≠ B[j]
            test = FALSE
            Break
        EndIf
    EndFor
    If test == TRUE
        Return TRUE
EndFor
Return FALSE
```

This algorithm uses nk steps therefore it is $\Theta(n^2)$. So it is in P.

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CENG 491 - Formal Languages and Automata

Name-Surname: 25.12.2014

ID Number:

CLASSWORK 8

You are given n distinct positive integers where $n \geqslant 3$. You want to find the third largest integer.

Show that this problem is in P.

Answer:

```
INPUT A[n]
For i = 1 to 3
    max = A[1]
     For j = 2 to n
         If A[j] > max
             max = A[j]
             index = j
         EndIf
     EndFor
     A[index] = 0
EndFor
Return max
```

This algorithm uses $3n$ steps therefore it is $\Theta(n)$. So it is in P.

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CENG 491 - Formal Languages and Automata

Name-Surname: 24.12.2014

ID Number:

CLASSWORK 9

The SHORTEST PATH problem is defined as follows:

You are given a weighted undirected graph G, and nodes s and t in the graph and a number W. Is there a path from s to t with weight $\leqslant W$?

Show that this problem is in NP.

Answer:

A given solution contains $n-1$ edges. We can check each edge in $n-1$ steps if each vertex has its list of edges. Then we have to check the sum. Therefore we do $(n-1)^2+n-1$ operations, so verifying algorithm is $\Theta(n^2)$.

This is polynomial, so the problem is in NP.

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Name-Surname: 25.12.2014

ID Number:

CLASSWORK 9

The MINIMUM SPANNING TREE problem is defined as follows:

You are given a weighted undirected graph G and a number W. Is there a spanning tree with weight $\leqslant W$?

Show that this problem is in NP.

Answer:

A given solution contains $\Theta(n)$ edges. We can check each edge in $\Theta(n)$ steps if each vertex has its list of edges. (If we have an unsorted list of edges, this will require n^2 operations but still, it is polynomial) Then we have to check the sum and also we have to check that all vertices are connected.

Verifying algorithm is $\Theta(n^2)$.

This is polynomial, so the problem is in NP.

Prepared by: Dr. Emre Sermutlu (2015)